Compositional Semantics

Propositional Logic

Ex. 1.

Among the following expressions, which are well-formed formulae of L_p?

(1)	$\neg(\neg P \lor Q)$	(5)	$(P \to ((P \to Q)))$	(9)	$(P \lor (Q \lor R))$
(2)	$P \lor (Q)$	(6)	$((P \to P) \to (Q \to Q))$	(10)	$-P \lor Q \lor R$
(3)	$\neg(Q)$	(7)	$((P_{28} \to P_3) \to P_4)$	(11)	$(\neg P \lor \neg\neg P)$
(4)	$(P_2 \to (P_2 \to (P_2 \to P_2)))$	(8)	$(P \to (P \to Q) \to Q)$	(12)	$(P \lor P)$

Ex. 2.

Show that for any φ, ψ and χ, the following pairs of formulae are logically equivalent:

(1)	$\neg\neg\varphi$	φ	
(2)	$\varphi \to \psi$	$\neg\varphi \lor \psi$	
(2')	$\varphi \to \psi$	$\neg(\varphi \land \neg\psi)$	
(3)	$\varphi \to \psi$	$\neg\psi \to \neg\varphi$	contraposition
(4)	$\varphi \leftrightarrow \psi$	$(\varphi \to \psi) \land (\psi \to \varphi)$	
(5)	$\varphi \leftrightarrow \psi$	$(\varphi \land \psi) \lor (\neg\varphi \land \neg\psi)$	
(6)	$\varphi \lor \varphi$	φ	idempotence
(7)	$\varphi \land \varphi$	φ	"
(8)	$\varphi \lor \psi$	$\psi \lor \varphi$	commutativity
(9)	$\varphi \land \psi$	$\psi \land \varphi$	"
(10)	$\varphi \lor (\psi \lor \chi)$	$(\varphi \lor \psi) \lor \chi$	associativity
(11)	$\varphi \land (\psi \land \chi)$	$(\varphi \land \psi) \land \chi$	"
(12)	$\varphi \land (\psi \lor \chi)$	$(\varphi \land \psi) \lor (\varphi \land \chi)$	distributivity
(13)	$\varphi \lor (\psi \land \chi)$	$(\varphi \lor \psi) \land (\varphi \lor \chi)$	"
(14)	$\neg(\varphi \land \psi)$	$\neg\varphi \lor \neg\psi$	de Morgan's law
(15)	$\neg(\varphi \lor \psi)$	$\neg\varphi \land \neg\psi$	"

Ex. 3.

Translate, as precisely as possible, the following sentences into propositional logic. Indicate to which sentence corresponds each propositional variable.

(1)
- a. The engine is not noisy, but it uses lots of gas.
- b. It is not the case that Max comes if Pam or Sam comes.
- c. John is not only stupid, he is also mean.
- d. I go to the beach or to the movies by foot or by car.
- e. John will come only if Paul doesn't come.

Ex. 4.

After having translated into propositional logic each sentence, show with a truth table
a. that (2a) logically implies (2b), and
b. that (3a) and (3b) are logically equivalent.

(2)
- a. Peter and Marie came, whereas Paul didn't.
- b. It is not the case that Paul came.

(3)
- a. For the company to start over, the CEO has to be replaced.
- b. Neither is the company starting over, nor has the CEO been replaced

Ex. 5.

Among the following discourses, which ones correspond to valid deductions?

(4)
- a. If Peter lied, then John is guilty. Yet John is guilty. Therefore Peter didn't lie.
- b. If Peter lied, then John is guilty. But Peter didn't lie. Therefore John isn't guilty.
- c. If Horacio loves Juliette, she will marry him. If Horacio doesn't love Juliette, she'll marry Gandalf. Yet Juliette will not marry Horacio, therefore she will marry Gandalf.
- d. If Horacio loves Juliette, she will marry him. If Horacio doesn't love Juliette, she'll marry Gandalf. Yet Juliette will marry Gandalf, therefore she won't marry Horacio.