Trivalent Semantics

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1 Linguistic Motivations

1.1 Presuppositions

(1) a. Mary stopped smoking.
 b. Mary didn't stop smoking.
 c. If Mary stopped smoking, he coughs less.
 $\rightsquigarrow$ Mary has smoked.

(2) a. Mary met John's son.
 b. Mary didn't meet John's son.
 c. If Mary met John's son, she talked to him.
 $\rightsquigarrow$ John has a unique son.

Exercise

Prove that, in propositional logic, if ϕ and ψ are such that:
$\phi \models \psi$ and $\neg\phi \models \psi$, then $\vDash \psi$ (i.e. ψ is a tautology)

The sentences in (1) and in (2) are said to *presuppose*, respectively, that Mary has smoked and that John has a unique son. Since 'Mary has smoked' is not (intuitively) tautological, presuppositions must be something other than classical entailment.

Intuition that goes back at least to Strawson: presuppositional sentence are sentenes that have no truth-value if a certain condition is not met. Classical example: *The king of France is bald.* Strawson argued that this sentence is neither true nor false.

1.2 Vague predicates

(3) Isabelle is tall.

The theshold for 'tallness' is vague. We may know everything there is to know about the world and not be sure whether (3) counts as true. So here the idea is that if Isabelle's height is a 'borderline case', then (3) is neither *clearly* true not *clearly* false.

2 Weak Kleene Semantics

Given a sentence ϕ, if any sentence ψ occuring in ϕ has the undefined truth-value, then ϕ has the undefined truth-value.

(4) John is tall and Mary is tall.

ϕ	$\neg\phi$
1	0
0	1
#	#

A	B	A $\wedge$ B
1	1	1
1	0	0
1	#	#
0	1	0
0	0	0
0	#	#
#	1	#
#	0	#
#	#	#

A	B	A $\vee$ B
1	1	1
1	0	1
1	#	#
0	1	1
0	0	0
0	#	#
#	1	#
#	0	#
#	#	#

A	B	A $\rightarrow$B
1	1	1
1	0	0
1	#	#
0	1	1
0	0	1
0	#	#
#	1	#
#	0	#
#	#	#

Suppose that John is very short, and Mary is borderline-tall. Then it seems we want to say that (4) is false. But according to Weak Kleene Semantics, if # is assigned to borderline-cases, (4) counts as false.

(5) a. Mary has a unique son, and Peter has met her son.
b. Mary has a unique son, but Peter hasn't met her son.

Do we want to say that (5) presupposes that Mary has a son, i.e. is not false, but undefined, if it turns out she doesn't have a son?

(6) It's not true that Mary has a unique son and that I have met her son.

(7) a. Mary used to smoke, and she has stopped smoking.
b. Mary used to smoke, and she hasn't stopped smoking.

(8) It's not true that Mary used to smoke and has stopped.

3 Strong Kleene Semantics

Idea: treat # as representing *ignorance* of truth-value, and check whether the truth-value of a complex formula can still be computed despite being uncertain about the truth-values of some subformulae.

ϕ	$\neg\phi$
1	0
0	1
#	#

A	B	A $\wedge$ B
1	1	1
1	0	0
1	#	#
0	1	0
0	0	0
0	#	0
#	1	#
#	0	0
#	#	#

A	B	A $\vee$ B
1	1	1
1	0	1
1	#	1
0	1	1
0	0	0
0	#	#
#	1	1
#	0	#
#	#	#

A	B	A $\rightarrow$B
1	1	1
1	0	0
1	#	#
0	1	1
0	0	1
0	#	1
#	1	1
#	0	#
#	#	#

- Deriving Strong Kleene tables from bivalent tables

Let f be a binary standard boolean function. Then f', the Strong Kleene version of f, is defined as follows.

(9) a. For any pair (x, y) of elements of $\{0, 1, \#\}$, a bivalent repair of (x, y) is any pair (x', y') of elements of $\{0, 1\}$ such that $x' = x$ if $x = 0$ or $x = 1$ and $y' = y$ if $y = 0$ or $y = 1$

b. $f'(x, y) = \begin{cases} 0 & \text{if for every bivalent repair } (x', y') \text{ of } (x, y), f(x', y') = 0 \\ 1 & \text{if for every bivalent repair } (x', y') \text{ of } (x, y), f(x', y') = 1 \\ \# & \text{otherwise} \end{cases}$

- Another way of looking at Strong Kleene

Call the third truth-value $\frac{1}{2}$. Then we can keep the previous semantic rules:

(10) A valuation is a function v from sentences to a truth-value in $\{0, \frac{1}{2}, 1\}$ such that, for any two formulae ϕ and ψ:

a. $v(\neg\phi) = 1 - v(\phi)$
b. $v(\phi \wedge \psi) = min(v(\phi), v(\psi))$
c. $v(\phi \vee \psi) = max(v(\phi), v(\psi))$
d. $v(\phi \rightarrow \psi) = max(1 - v(\phi), v(\psi))$

4 Empirical Discussion

4.1 Presupposition

4.1.1 Some facts about presupposition projections

Notation. We write ϕ_p to indicate that ϕ presupposes p.

Fact #1. A sentence of the form $\phi \wedge \psi_p$ presupposes $\phi \rightarrow p$

(11) a. France is a monarchy and the king of France is bald
b. It is not the case that France is a monarchy and that the king of France is bald
$\rightsquigarrow$ France is a monarchy $\rightarrow$ there is a unique king of France
$\rightsquigarrow$ Presupposition is satisfied automatically (is tautological)

(12) a. Mary lives in the countryside and her garden is big
b. It is not the case that Mary lives in the countryside and that her garden is big
$\rightsquigarrow$ If Mary lives in the countryside, she has a garden

(13) Context: every doctor is rich
a. Mary is a doctor but Peter does not know that she is rich
b. It is not the case that Mary is a doctor but that Peter does not know that she is rich $\rightsquigarrow$ If Mary is a doctor, she is rich (contextual tautology)

(14) a. Mary's elephant is nice
$\rightsquigarrow$ infelicitous unless it is known that Mary has an elephant
b. Mary is French and her elephant is nice
$\rightsquigarrow$ felicitous if it is known that every French person owns an elephant.

(15) a. John's instrument is well hidden
b. John plays the violiin and his instrument is well hidden.
c. Is John's instrument well hidden
d. Is it true that John plays the violin and his instrument is well hidden?

- Proviso Problem. Out of the blue, (14b) triggers the inference that Mary owns an elephant (and not simply that if she is French, she owns an elephant).

Fact #2. A sentence of the form 'If ϕ, then ψ_p' presupposes $\phi \rightarrow p$

(16) If France is a monarchy, the king of France is bald.

(17) If Mary lives in the countryside, her garden is big
$\rightsquigarrow$ If Mary lives in the countryside, she has a garden

(18) If Mary is a doctor, Peter knows that she is rich
$\rightsquigarrow$ If Mary is a doctor, she is rich

(19) If Mary is French, her elephant is nice
$\rightsquigarrow$ If Mary is French, she has an elephant.

Fact #3 (more debated) A sentence of the form $\phi \vee \psi_p$ presupposes $\neg\phi \to p$

(20) France is not a monarchy, or the king of France is bald

(21) This house has no bathroom, or its bathroom is well hiden.

(22) Mary isn't a doctor, or Peter doesn't know that she is rich

(23) John does not own a violin, or his instrument is well hidden

4.2 Strong Kleene predicts these facts

Assumption: the presupposition of a complex sentence ϕ is the proposition that is true if and only if ϕ is not undefined.

ϕ	ψ_p	p	$\phi \wedge \psi_p$
1	1	1	1
1	0	1	0
1	#	0	#
0	1	1	0
0	0	1	0
0	#	0	0

ϕ	ψ_p	p	$\phi \to \psi_p$
1	1	1	1
1	0	1	0
1	#	0	#
0	1	1	1
0	0	1	1
0	#	0	1

- When are $\phi \wedge \psi_p$ and $\phi \to \psi_p$ undefined? Just in case ϕ is true and p is false. So $\phi \wedge \psi_p$ has a defined truth-value if either ϕ is false or p is true, i.e. if the material conditional $\phi \to p$ is true.
- Since a sentence's presupposition can be equate with the conditions under which it has a defined truth-value, both $\phi \wedge \psi_p$ and $\phi \to \psi_p$ are predicted to presuppose $\phi \to p$.

4.3 Vagueness

(24) Either Peter or Mary is tall.
$\rightsquigarrow$ Clearly true as soona s one of them is clearly tall, but clearly false only if both are clearly not tall.