Formal Languages applied to Linguistics

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Overview

1. Formal Languages
2. Formal Grammars
3. Regular Languages
   - Definition
   - Automata
   - Properties
4. Formal complexity of Natural Languages

Definition

3 possible definitions

1. a regular language can be generated by a regular grammar
2. a regular language can be defined by rational expressions
3. a regular language can be recognized by a finite automaton

Def. 15 (Rational Language)

A rational language on Σ is a subset of Σ^* inductively defined thus:

- $\emptyset$ and $\{\varepsilon\}$ are rational languages ;
- for all $a \in X$, the singleton $\{a\}$ is a rational language ;
- for all g and h rational, the sets $g \cup h$, $g.h$ and g^* are rational languages.

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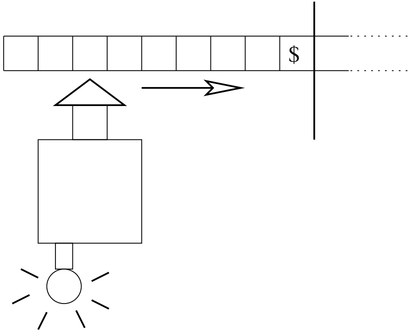
Metaphoric definition



Formal definition

Def. 16 (Finite deterministic automaton (FDA))

A finite state deterministic automaton $\mathcal{A}$ is defined by :

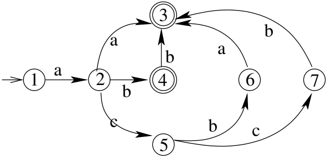
$$\mathcal{A} = \langle Q, \Sigma, q_0, F, \delta \rangle$$

- Q is a finite set of states
- Σ is an alphabet
- q_0 is a distinguished state, the initial state,
- F is a subset of Q, whose members are called final/terminal states
- δ is a mapping **fonction** from $Q \times \Sigma$ to Q. Notation $\delta(q, a) = r$.

Example

Let us consider the (finite) language $\{aa, ab, abb, acba, accb\}$.
The following automaton recognizes this langage: $\langle Q, \Sigma, q_0, F, \delta \rangle$, avec $Q = \{1, 2, 3, 4, 5, 6, 7\}$, $\Sigma = \{a, b, c\}$, $q_0 = 1$, $F = \{3, 4\}$, and δ is thus defined:

δ :
(1,a) $\mapsto$ 2
(2,a) $\mapsto$ 3
(2,b) $\mapsto$ 4
(2,c) $\mapsto$ 5
(4,b) $\mapsto$ 3
(5,b) $\mapsto$ 6
(5,c) $\mapsto$ 7
(6,a) $\mapsto$ 3
(7,b) $\mapsto$ 3



	a	b	c
$\rightarrow$ 1	2		
2	3	4	5
$\leftarrow$ 3			
$\leftarrow$ 4		3	
5		6	7
6	3		
7		3	

Recognition

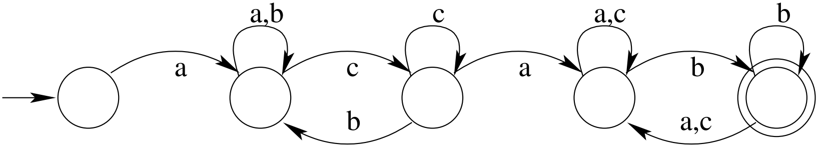
Recognition is defined as the existence of a sequence of states defined in the following way. Such a sequence is called a path in the automaton.

Def. 17 (Recognition)

A word $a_1a_2...a_n$ is **recognized/accepted** by an automaton iff there exists a sequence $k_0, k_1, ..., k_n$ of states such that:

$$k_0 = q_0$$
$$k_n \in F$$
$$\forall i \in [1, n], \;\; \delta(k_{i-1}, a_i) = k_i$$

Example

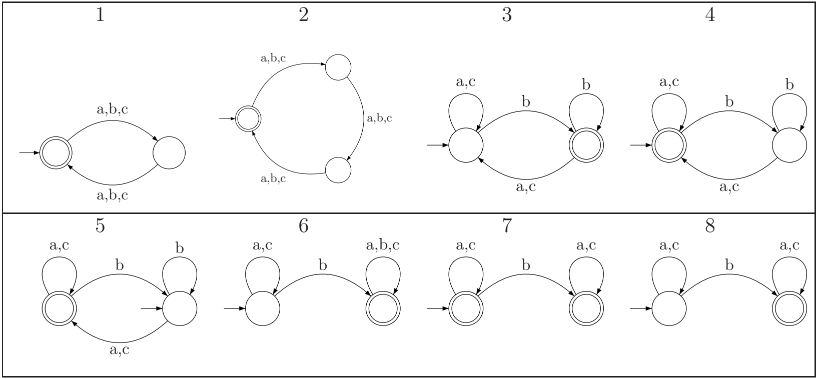


Exercices

Let $\Sigma = \{a, b, c\}$. Give deterministic finite state automata that accept the following languages:

1. The set of words with an even length.
2. The set of words where the number of occurrences of b is divisible by 3.
3. The set of words ending with a b.
4. The set of words not ending with a b.
5. The set of words non empty not ending with a b.
6. The set of words comprising at least a b.
7. The set of words comprising at most a b.
8. The set of words comprising exactly one b.

Answers


1
a,b,c
a,b,c
2
a,b,c
a,b,c
a,b,c
3
a,c
b
b
a,c
4
a,c
b
b
a,c
5
a,c
b
b
a,c
6
a,c
b
a,b,c
7
a,c
b
a,c
8
a,c
b
a,c