# Compositionality 

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## Overview

Frege's principle

```
Typed }\lambda\mathrm{ -calculus
    Type theory
    Montague's language
```

First fragment

## Frege's principle

The meaning of an expression is uniquely determined by the meanings of its parts and their mode of combination.

Motivation : Humbolt's view on finite means for infinite sentences

## Consequences

- Locality principle : lexical items have a meaning that is independant of the expression they occur in.
- Substitution principle : synonymous expressions may be substituted for each other without changing the meaning of the complex expression in which they occur.
- Parts of well formed sentences have «meaning»
- Meanings can be «composed »: Frege's saturation idea
$\lambda$-terms can represent individual meanings and functional application can represent semantic composition.


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## Type theory

1. $e$ is a type
2. $t$ is a type
3. if $a$ and $b$ are types, then $\langle a, b\rangle$ is a type

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- $D_{e}=A$
- $D_{t}=\{0,1\}$
- $D_{\langle a, b\rangle}=$ the set of mappings from $D_{a}$ to $D_{b}$.


## Meaningful expressions

For $a, b$ types :

- variables and individual constants of type a belong to $M E_{a}$.
- if $\alpha \in M E_{\langle a, b\rangle}$ and $\beta \in M E_{a}$ then $(\alpha) \beta \in M E_{b}$.
- if $u$ is a variable of type $a$ and $\alpha \in M E_{b}$, then $\lambda u . \alpha \in M E_{\langle a, b\rangle}$.
- if $\varphi$ and $\psi$ are in $M E_{t}$, then the following expressions are also in $M E_{t}: \neg \varphi,(\varphi \wedge \psi),(\varphi \vee \psi),(\varphi \rightarrow \psi)$.
- if $\varphi$ is in $M E_{t}$ and $u$ is a type a variable, then $\forall u \varphi$ and $\exists u \varphi$ are in $M E_{t}$.


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