Formal Languages and Linguistics

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Exercises I

1. Let g be the grammar $S \rightarrow Sa$
   $S \rightarrow a$
   Describe informally the language of this grammar.
2. Propose a grammar that generates words composed of any number of as followed by exactly one b.
3. Remember that Dyck language is engendered by the grammar
   $S \rightarrow (S)$
   $S \rightarrow \varepsilon$
   Give a grammar such that evry word has exactly two ending parenthesis for every opening parenthesis, while remaining well balanced.
4. Give a grammar such that evry word has as many opening parenthesis than closing parenthesis.

Exercises II

5. Modify the grammar $E \rightarrow E + E$; $E \rightarrow 1 \mid 2 \mid 3$ in such a way that « (2+3)+1 » is part of its language.
6. Show that the grammar $E \rightarrow E + E$; $E \rightarrow E \times E$; $E \rightarrow 1 \mid 2 \mid 3$ is ambiguous.
7. Show that the grammar $E \rightarrow E + T \mid T, T \rightarrow T \times F \mid F, F \rightarrow (E) \mid a$ is **not** ambiguous.
8. Give a contex-free grammar that generates each of the following languages (alphabet $\Sigma = \{a, b, c\}$).
   - $L_0 = \{w \in X^* \ / \ w = a^n \ ; \ n \geq 0\}$
   - $L'_0 = \{w \in X^* \ / \ w = a^n b^n ca \ ; \ n \geq 0\}$
   - $L_1 = \{w \in X^* \ / \ w = a^n b^n c^p ; n > 0 \text{ et } p > 0\}$
   - $L_2 = \{w \in X^* \ / \ w = a^n b^n a^m b^m ; n, m \geq 1\}$

Exercises III

- $L'_3 = \{w \in X^* \ / \ |w|_a = |w]_b\}$
- $L_3 = \{w \in X^* \ / \ |w|_a = 2|w]_b\}$
- $L_4 = \{w \in X^* \ / \ \exists x \in X^* \text{ tq } w = x\overline{x}\}$
- $L_5 = \{w \in X^* \ / \ w = \overline{w}\}$

9. For each of the following grammars, give the generated language, and the type they have in Chomsky's hierarchy.

$$\begin{array}{lcl} S & \rightarrow & S_1 S_2 \\ S_1 & \rightarrow & aS_1 b \mid ab \\ S_2 & \rightarrow & cS_2 \mid c \end{array}$$

$$\begin{array}{lcl} S & \rightarrow & aSBC \\ S & \rightarrow & aBC \\ CB & \rightarrow & BC \\ aB & \rightarrow & ab \\ bB & \rightarrow & bb \\ bC & \rightarrow & bc \\ cC & \rightarrow & cc \end{array}$$