The OT error-driven ranking model of the acquisition of phonotactics

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Phonotactics

- Knowledge of the phonotactics of a language is knowledge of the distinction between licit and illicit sounds and sound combinations
- For instance, English speakers know that *blik* could be a licit English word while *bnik* could not, although both are unattested
- Knowledge of alternations is knowledge of how to repair illicit sound combinations

The acquisition of phonotactics

Phonotactics is acquired:

- **Early**: nine-month-old infants already react differently to licit and illicit sound combinations (Jusczyk *et al.* 1993)
- **Before morphology**: namely at a stage where the child has still no access to phonological alternations (Hayes 2004)
- **Gradually**: as the target adult phonotactics is approached through a stepwise progression of intermediate stages (Mc Leod *et al.* 2001)

2:3	**2:5**	**2:6**	**2:8**
tʌk	lʌk	dk	flʌk
tʌk	lʌk	dʌk	θlʌk
	flʌkθ	klʌkθ	θlʌk
	klʌkθ		

2:8	**2:10**	**2:11**	**3:1**
kʌk	kəlʌːk	klʌk	klʌk
kʌk	kʌk	klʌk	klʌk
kʌk	kəlʌk	klʌks	
	kəlʌk	kʌk	

Want a model of the child acquisition of phonotactics that makes sense of these properties

Optimality Theory (OT)

OT is built on three core ideas (Prince & Smolensky 2004, Kager 1999):

- Relevant phonological properties are extracted by a set of universal, innate constraints, that measure how structures deviate from the ideal
  - **Markedness constraints** measure how phonological structures violate wellformedness conditions:

    *DORSAL is violated by dorsal consonants (i.e. by [kl] and [k])
  - **Faithfulness constraints** measure how phonological realizations differ from intended targets:

    MAX is violated by deletion (i.e. by [k] as the realization of target /kl/)
- Two or more constraints can conflict
  - MAX prefers [kl] over [t] as the production of the target /kl/
  - *DORSAL prefers [t] over [kl] instead
- Grammars differ in how they rank constraints and conflicts are resolved by a grammar in favor of the constraint it ranks at the top

The EDRA model of the acquisition of phonotactics

- **Initialization**: Markedness constraints start at the top, faithfulness constraints at the bottom, yielding the smallest language
- **Loop**: maintains a numerical representation of the current ranking by assigning each constraint a ranking value reflecting its relative rank:

  Step 1 receives a piece of data from the incoming stream of data
  - receives the word *clock*, that shows that /kl/ is a licit cluster

  Step 2 checks whether its current ranking accounts for this datum
  - checks whether it accounts for the mapping /kl/→[kl]
  - suppose it instead predicts [kl] to be unavailable because reduced to [t]

  Step 3 if it makes a mistake, it updates the current ranking
  - slightly demotes *DORSAL that prefers the loser mapping /kl/→[t]
  - slightly promotes MAX, that prefers the winner mapping /kl/→[kl]
- **Termination**: loops until constraints (hopefully) intersperse in a ranking consistent with the target phonotactics

Examples: Tesar & Smolensky's (1998) EDCD, Boersma's (1998) GLA.

An example: modeling the acquisition of syllable types

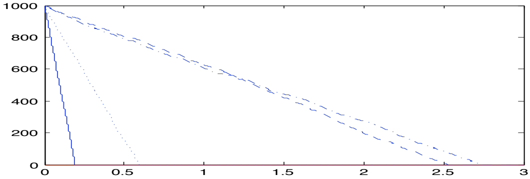
- Attested acquisition paths for Dutch learners (Levelt *et al.* 2001):

$$\text{CV} \Rightarrow \text{CVC} \Rightarrow \{\text{V}, \text{VC}\} \begin{array}{l} \nearrow \{\text{CCV}, \text{CCVC}\} \Rightarrow \{\text{CVCC}, \text{VCC}, \text{CCVCC}\} \\ \searrow \{\text{CVCC}, \text{VCC}\} \Rightarrow \{\text{CCV}, \text{CCVC}, \text{CCVCC}\} \end{array}$$

- OT constraints for syllable types (Prince & Smolensky 2004):

NoCoda Onset CompCoda CompOnset Faith

- Re-ranking rule (a mixture of EDCD and GLA):
  - Demote by 1 loser-preferring constraints
  - do nothing to winner-preferring constraint
- Ranking dynamics matches acquisition order (Boersma&Levelt 2001):



Strengths of the OT error-driven model

EDRAs have been endorsed by the OT acquisition literature because

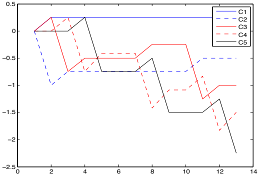
- **Gradualness**: intermediate rankings entertained by EDRAs on the way to the final grammar correspond to intermediate child acquisition stages, thus modeling gradualness
- **Morphology-free**: being trained on faithful mappings (/kl/→[kl]), EDRAs only looks at surface phonology and don't require alternations, that become available only later on, when morphology kicks in
- **Memory-free**: EDRAs don't keep track of previously seen data, and thus don't impose unrealistic memory requirements (contrary to batch models, that are allowed to glimpse at the entire set of data at once).

Main goal of my current research:

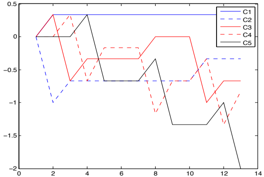
show that EDRAs provide a proper model of the acquisition of phonotactics, both from a computational and a modeling perspective

A combined computational/modeling approach

- Establishing the adequacy of EDRAs looks like an empirical modeling issue: you test them on lots of data, as Levelt's syllable types data
- Empirical it is indeed. Yet, here is a way to appreciate the importance of a solid computational understanding of the model:



promote by $p - \epsilon$



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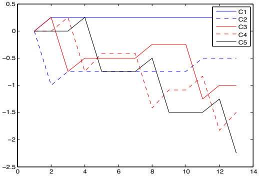

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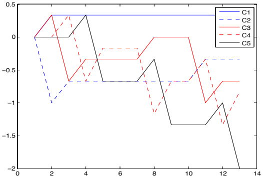
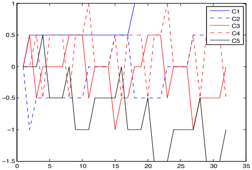


promote by p



promote by $p + \epsilon$

- We need an integrated computational/modeling approach:
  - individuate specific computational desiderata
  - develop implementations of EDRAs that provably satisfy them
  - test the modeling adequacy of these EDRAs on child acquisition data
- This project contributes to a new field of linguistic research, called Cognitive Computational Phonology

Core issues of the theory of OT error-driven learning

- Issue #1 does the model eventually stop making mistakes and settle on a final grammar? (**convergence**)
- Issue #2 does the final grammar entertained by the model indeed capture the target phonotactics? (**correctness**)
- Issue #3 do the learning sequences predicted by the model match attested child acquisition paths? (**modeling adequacy**)
- Issue #4 how does the model behave in the presence of noise and how can it make sense of child variation? (**robustness/variation**)
- Issue #5 how can the choice of the OT framework be justified from a learning theoretic perspective? (**framework selection**)

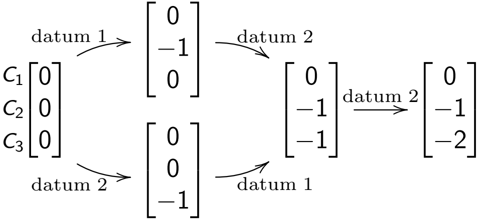
The issue of convergence

- The most basic computational desiderata on EDRAs is:
  - Convergence: is it the case that the EDRA can only make a finite number of mistakes (when trained on consistent data)?
  - Efficiency: is it the case that the number of mistakes grows slowly with the complexity of the underlying typology (= number of constraints)?
- For simplicity, assume from now on that each piece of data has a unique loser-preferrer (this is not a restrictive assumption)
- The prototypical EDRA re-ranking rule works as follows:
  - Demotion component: decrease the ranking value of the unique loser-preferrer by a certain amount, say 1 for concreteness
  - Promotion component: increase the ranking value of each one of the winner-preferrers by a certain promotion amount, call it $p \geq 0$
- The crucial issue is how to choose the promotion amount p:
  - which choices are needed for efficient convergence?
  - do they yield good modeling predictions?

The choice $p = 0$: Tesar & Smolensky's (1998) analysis

- **Idea**: to be on the safe side, we only perform demotion
  - demote the loser-preferrer by 1
  - do not promote any constraints
- **Example**:

$$\begin{array}{c} \text{datum 1} \\ \text{datum 2} \end{array} \overset{\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}}{\begin{bmatrix} \text{W} & \text{L} & \text{W} \\ e & \text{W} & \text{L} \end{bmatrix}}$$



- **Sketch of T&S's analysis**:
  - C_1 stays at 0; C_2 never makes it below -1; C_3 never below -2
  - At each update, one constraint is demoted
  - As constraints cannot drop too much, then you can't update too much
  - I.e., the number of updates is at most $0 + 1 + 2 = \frac{1}{2}n(n-1)$, $n = 3$

The choice $p = 0$: Tesar & Smolensky's analysis (cont'd)

- Each set of OT-compatible data has the shape represented here, by reordering the data and possibly relabeling the constraints
- C_1 stays at 0, C_2 never makes it below -1, C_3 never below -2, ..., C_n never below $n-1$
- At each update, one constraint is demoted
- As constraint cannot drop too much, then you can't update too much
- I.e., the number T of updates is at most

$$\begin{array}{ccccccc} C_1 & C_2 & \dots & \dots & \dots & \dots & C_n \end{array}$$

$$\left[\begin{array}{cccccc}
\text{W} & & & & & \\
\vert & \dots & \dots & \dots & \dots & \dots \\
\text{W} & & & & & \\
\hline
e & \text{W} & & & & \\
\vert & \vert & \dots & \dots & \dots & \dots \\
e & \text{W} & & & & \\
\hline
 & & \ddots & \dots & \dots & \dots \\
\hline
e & e & - & e & \text{W} & \\
\vert & \vert & & \vert & \vert & \dots \\
e & e & - & e & \text{W} &
\end{array}\right]$$

$$\begin{aligned} T &\le \begin{array}{c}\#\text{ of times} \\ C_1 \text{ is demoted}\end{array} + \begin{array}{c}\#\text{ of times} \\ C_2 \text{ is demoted}\end{array} + \cdots + \begin{array}{c}\#\text{ of times} \\ C_n \text{ is demoted}\end{array} \\ &\le 0 + 1 + \ldots + (n-1) = \frac{1}{2}n(n-1) \end{aligned}$$

Theorem

T&S's demotion-only EDRA performs at most $\frac{1}{2}n(n-1)$ updates (where n is the number of constraints)

But we want constraint promotion!

- The model always assumes the winner mapping is the faithful one (/kl/ $\rightarrow$ [kl]), which is the safe option if you are blind to alternations
- This means the faithfulness constraints are never loser-preferrers and thus are never re-ranked by T&S's demotion-only EDRA
- This cannot be right:
  - the demotion-only EDRA fails on target languages that require a different relative ranking of the faithfulness constraints
  - the demotion-only EDRA fails to model learning paths where the repair changes over time, requiring re-ranking of the faithfulness constraints
- Thus constraint promotion is needed from a modeling perspective
- Unfortunately, constraint promotion is not easy to get from a computational perspective: Tesar & Smolensky warn us against it!

The choice $p = 1$: Boersma's (1998) attempt

- **Idea**: if we can't choose which to promote, let's just promote all
  - Demote the loser-preferrer by 1
  - promote each winner-preferrer by 1
- But Pater (2008) reports simulation results with the following data sampled uniformly where the EDRA does not converge

$$\begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array}$$
$$\left[\begin{array}{ccccc} \mathrm{W} & \mathrm{L} & \mathrm{W} & & \\ & \mathrm{W} & \mathrm{L} & \mathrm{W} & \\ & & \mathrm{W} & \mathrm{L} & \mathrm{W} \\ & & & \mathrm{W} & \mathrm{L} \end{array}\right]$$

- Actually, the EDRA never converges on these data, no matter how the data are sampled
- How could we reconcile these two opposite perspectives?
  - the modeling perspective, that wants constraint promotion
  - the computational perspective, that wants demotion-only

The choice $p < 1/w$: calibrated constraint promotion

- **Idea**: calibrate promotion, so as not to disrupt convergent demotion
  - Demote the loser-preferrer by 1
  - promote each of the w winner-preferrers by less than $\frac{1}{w}$, say by $\frac{1}{w+1}$
- **Step 1 of the analysis**:
  - C_1 stays at 0, C_2 never makes it below -1, C_3 never below -2, ...
  - hence the sum of current ranking values is always at least $-\frac{1}{2}n(n-1)$
- **Step 2 of the analysis**:
  - Suppose the current datum has $w = 3$ winner-preferrers
  - thus the sum of the ranking values is decreased by 1 (demotion)
  - and increased three times by $\frac{1}{w+1} = \frac{1}{4}$ (promotion)
  - overall it is therefore decreased by $\frac{1}{4} = \frac{1}{w+1} \geq \frac{1}{n}$
- **Step 3 of the analysis**: updates cannot go on for ever, as the sum of the ranking values shrinks with every update but cannot get too small

Theorem

The well calibrated promotion/demotion EDRA performs at most $\frac{1}{2}n^2(n-1)$ updates (where n is the number of constraints)

The choice $p = 1/w$: the breakpoint

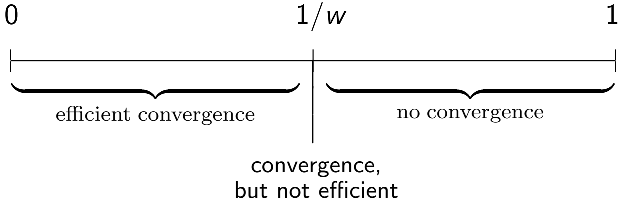
- **Idea**: what if we promote by the smallest non calibrated amount?
  - Demote the loser-preferrer by 1
  - promote each of the w winner-preferrers by exactly $\frac{1}{w}$
- The previous analysis of convergence does not extend to this case:
  - Suppose the current datum has $w = 3$ winner-preferrers
  - thus the sum of the ranking values is decreased by 1 (demotion)
  - and increased three times by $\frac{1}{w} = \frac{1}{3}$ (promotion)
  - overall, the sum of the ranking values remains constant
- Alternative analysis I: use convergence of the Perceptron algorithm
- Alternative analysis II:
  - suppose the data are OT-compatible
  - this entails that the update vectors are conically independent
  - this entails in turn that the EDRA cannot loop
  - and this entails in turn that the EDRA converges

Theorem

The EDRA with $p = \frac{1}{w}$ converges. Yet, efficiency is lost, as the worst-case number of mistakes grows exponentially in the number n of constraints.

Conclusion on convergence

- From a modeling perspective, we need EDRAs that perform constraint promotion too
- From a computational perspective, efficiently convergent constraint promotion is possible through proper calibration



- ⟹OT grammars must be represented by the learner as numerical ranking values not as combinatorial rankings, as calibration of constraint promotion requires a numerical representation of rankings

The issue of correctness

- Phonotactics is the knowledge of licit vs illicit sound combinations
- A grammar (i.e. an OT ranking) is called
  - consistent, provided it rules in every licit form
  - restrictive, provided it also rules out every illicit form
  - correct, provided it is both consistent and restrictive
- Now let's look at EDRAs:
  - if it converges, then its final grammar is consistent
  - yet, it could be non-restrictive (e.g. all faithfulness constraints at top)
  - a convergent EDRA is correct provided that the final grammar entertained at converge is also restrictive, and thus correct
- Correctness is a pressing issue for error-driven learning:
  - we only have control on the initial ranking and the re-ranking rule
  - thus, the acquisition path is governed by the stream of data
  - so that the model behaves as a leaf in the wind of data
  - and there seem to be no reasons to expect correctness
- I want to show that this pessimism is not warranted

My research strategy on EDRA's correctness

- **Step 1 of the analysis**:
  - a language is called $\mathcal{F}$-irrelevant if the relative ranking of the faithfulness constraints does not matter for that language
  - EDRAs are always correct on $\mathcal{F}$-irrelevant languages, with no restrictions on the constraints
- **Step 2 of the analysis**:
  - The problem of the acquisition of phonotactics is not solvable in its full generality, without restrictions on constraints
  - Thus, correctness on $\mathcal{F}$-relevant languages cannot be achieved without restrictions on constraints (by EDRAs or by any other algorithm)
- **Step 3 of the analysis**: investigate the algorithmic implications of generalizations on phonologically plausible constraints for error-driven correctness on $\mathcal{F}$-relevant languages:
  - assemble a very large constraint set from the OT literature
  - extract $\mathcal{F}$-relevant languages from the corresponding typology
  - study behavior of EDRAs on those

$\mathcal{F}$-irrelevant languages

- **Informally**: a language is $\mathcal{F}$-irrelevant provided the relative ranking of the faithfulness constraints does not matter for that language
- **Formally**:
  - a partial ranking is a partial order of the constraint set: there might be two constraints that are not ranked relative to each other
  - a partial ranking generates a language provided each one of its total refinements generates that language (in the usual OT sense)
  - a language is $\mathcal{F}$-irrelevant provided it is generated by a partial ranking that doesn't rank any two faithfulness constraints relative to each other
- **Remarks**:
  - there are partial rankings that don't generate any language (as they admit total refinements that generate different languages)
  - certain ranking conditions are crucial for consistency (if you drop them, then you allow for total refinements that generate smaller languages)
  - certain ranking conditions are crucial for restrictivity (if you drop them, then you allow for total refinements that generate larger languages)

Examples

- The OT typology:

$$\textit{forms} = \left\{ \begin{array}{cccc} \text{ap} & \text{ab} & \text{as} & \text{az} \\ \text{apsa} & \text{absa} & \text{apza} & \text{abza} \end{array} \right\}$$

$$\textit{constraints} = \left\{ \begin{array}{cc} F_1 = \textsc{Ident}[\textsc{fric-voi}] & F_2 = \textsc{Ident}[\textsc{stp-voi}] \\ M_1 = *[+\textsc{fric-voi}] & M_2 = *[+\textsc{stp-voi}] \\ \multicolumn{2}{c}{M = \textsc{Agree}[\textsc{stp-voi},\ \textsc{fric-voi}]} \end{array} \right\}$$

- A partial ranking and some of its total refinements:

$$\begin{array}{cc} M & \\ | & \\ F_1 & F_2 \\ | & | \\ M_1 & M_2 \end{array} \quad \Rightarrow \quad \begin{array}{ccc} F_2 & M & M \\ M & F_2 & F_2 \\ M_2 & F_1 & M_2 \\ F_1 & M_2 & F_1 \\ M_1 & M_1 & M_1 \end{array}$$

- This partial ranking thus generates L, which is therefore $\mathcal{F}$-irrelevant:

$$L = \left\{ \begin{array}{cccc} \text{pa,} & \text{ba,} & \text{sa,} & \text{za,} \\ \text{apsa,} & & & \text{abza,} \end{array} \right\}$$

Examples (cont'd)

- Again, the same OT typology:

$$forms = \left\{ \begin{array}{cccc} \text{ap} & \text{ab} & \text{as} & \text{az} \\ \text{apsa} & \text{absa} & \text{apza} & \text{abza} \end{array} \right\}$$

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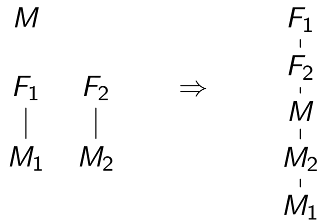
- The following partial ranking admits all preceding total refinements:



- But it also admits the total refinement above that generates the entire set of forms, so that this partial ranking generates nothing
- This shows that some ranking conditions are needed for restrictiveness

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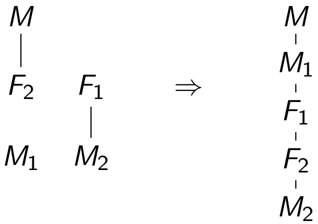


- But it also admits the total refinement above that is not consistent with the form [ab] and thus does not generate the language L
- This shows that some ranking conditions are needed for consistency

EDRAs get right the ranking of M above F

- Suppose that the target language
  - is $\mathcal{F}$-irrelevant, namely is generated by a partial ranking that does not rank faithfulness constraints relative to each other
  - and this partial ranking ranks some M above some F
- This entails that:
  - only markedness constraints can be ranked above M, not any faithfulness constraint
  - the top-ranked one(s) among the markedness constraints ranked above M is not violated by any form in the language

- We can thus reason as follows:
  - there can be at most $n - 2$ markedness constraints ranked above M
  - T&S thus guarantee that M cannot drop by more than $n - 3$
  - suppose the EDRA only demotes or else does not promote "too much"
  - so that it won't promote F that high from its initial low position
  - the EDRA will thus converge to a final ranking that enforces $M > F$
- This reasoning fails if M needs to be ranked underneath some other F', as could be the case if the language is not $\mathcal{F}$-irrelevant

EDRAs get right the ranking of M above M'

- Suppose that the target language
  - is generated by a partial ranking that ranks M above M'
  - and this ranking happens to be crucial in particular for restrictiveness
- This means that:
  - there is a form x not in the language
  - because reduced to some form y
  - and M is the "unprotected" L
  - while M' is a W that could protect M'
  - that thus needs to be ranked below M

$$\begin{array}{c} \\ /x/ \to [x], \\ /x/ \to [y]: \end{array} \begin{array}{ccccc} \ldots & M & \ldots & M' & \ldots \\ \Big[\ldots & \mathrm{L} & \ldots & \mathrm{W} & \ldots\Big] \end{array}$$

- Now we can reason as follows:
  - the form y must be in the language
  - suppose "candidacy" is symmetric
  - so that x is a candidate of y
  - now M' is an L that needs to be protected with the W of M
  - so that M needs to be ranked above M' also for consistency
  - as convergent EDRAs have no problems with consistency
  - the final grammar entertained by convergent EDRAs enforces $M > M'$

$$\begin{array}{c} \\ /y/ \to [y], \\ /y/ \to [x]: \end{array} \begin{array}{ccccc} \ldots & M & \ldots & M' & \ldots \\ \Big[\ldots & \mathrm{W} & \ldots & \mathrm{L} & \ldots\Big] \end{array}$$

Step 1 of correctness: $\mathcal{F}$-irrelevant languages

Summarizing:

F | F' — cannot matter, if the language is $\mathcal{F}$-irrelevant	M | F — EDRAs get it right, if the language is $\mathcal{F}$-irrelevant and promotion is null or calibrated
F | M — can only matter for consistency, so that convergent EDRAs get it right	M | M' — it cannot matter only for restrictiveness provided "candidacy" is symmetric, so that EDRAs get it right

Thus, we conclude that:

Theorem (Step 1 of the analysis of correctness)

If "candidacy" is symmetric:

- *demotion-only EDRAs are correct on any $\mathcal{F}$-irrelevant language*
- *the result extends to EDRAs that do not promote too much*
- *but not to EDRAs that promote too much (e.g. Boersma's GLA)*

Step 2 of correctness: $\mathcal{F}$-relevant languages

- Convergence can be achieved in full generality, namely with no restrictions whatsoever on the underlying OT constraint set
- Correctness on $\mathcal{F}$-irrelevant languages can be achieved in full generality too (only need "candidacy" to be symmetric)
- But correctness on $\mathcal{F}$-relevant languages provably cannot be achieved in full generality, no matter the algorithm we pick

Theorem (Step 2 of the analysis of correctness)

The problem of the acquisition of phonotactics in OT cannot be solved efficiently in its general formulation (it is NP-complete), namely without restrictive assumptions on the underlying OT constraint set

- Is it the case that phonologically plausible restrictions on constraints ensure correctness of EDRAs on $\mathcal{F}$-relevant languages?
- A positive answer would provide formidable support for EDRAs as a proper model of the child's acquisition of phonotactics

A couple of cases of segmental OT phonotactics

- Obstruent voicing (Lombardi 1999, Prince & Tesar 2004):
  - *forms* $= \left\{ \begin{array}{cccc} \text{ap} & \text{ab} & \text{as} & \text{az} \\ \text{apsa} & \text{absa} & \text{apza} & \text{abza} \end{array} \right\}$
  - *constraints* $=$
    $$\left\{ \begin{array}{cc} F_1 = \textsc{Ident}[\textsc{fric-voi}] & F_2 = \textsc{Ident}[\textsc{stp-voi}] \\ M_1 = *[+\textsc{fric-voi}] & M_2 = *[+\textsc{stp-voi}] \\ \multicolumn{2}{c}{M = \textsc{Agree}[\textsc{stp-voi},\ \textsc{fric-voi}]} \end{array} \right\}$$

- Laryngeal inventory (Hayes 2004):
  - *forms* $= \{ \ \text{t} \quad \text{d} \quad \text{t}^h \quad \text{d}^h \ \}$
  - *constraints* $= \left\{ \begin{array}{cc} F_1 = \textsc{Ident}[\textsc{voice}] & F_2 = \textsc{Ident}[\textsc{asp}] \\ M_1 = *[+\textsc{voice}] & M_2 = *[+\textsc{asp}] \\ \multicolumn{2}{c}{M = *[+\textsc{voice},\ +\textsc{asp}]} \end{array} \right\}$

Extracting a toy framework for OT segmental phonotactics

- There are two partial binary phonological features

$$\varphi_1, \varphi_2 : \textit{forms} \mapsto \{0, 1, \#\}$$

- Segments are pairs of feature values:

$$\mathbf{x} = \langle x_1, x_2 \rangle,\ x_i \in \{0, 1, \#\}$$

- Candidates obtained by changing feature values of in all possible ways
- The constraint set can contain:
  - *faithfulness constraint* F_i corresponding to feature φ_i

    $$F_i(\mathbf{x}, \mathbf{y}) = 1 \iff x_i \neq y_i$$

  - *(simple) markedness constraint* M_i corresponding to feature φ_i

    $$M_i(\mathbf{y}) = 1 \iff y_i = 1 = \text{the marked value}$$

  - *binary markedness constraint* M^μ with *markedness pattern* μ:

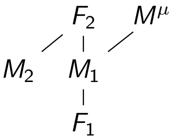
    $$M^\mu(\mathbf{y}) = 1 \iff \langle y_i, y_j \rangle \in \mu$$

- There are sixteen possible markedness patterns μ. Some examples:
  - $\mu = \{\langle 1, 1 \rangle\}$ — *[+VOICE, +ASP]
  - $\mu = \{\langle 0, 1 \rangle, \langle 1, 0 \rangle\}$ — AGREE[STP-VOI, FRIC-VOI]
  - $\mu = \{\langle 0, 0 \rangle, \langle 1, 1 \rangle\}$ — OCP-type constraint

Phonologically plausible binary markedness constraints

- A binary markedness constraint is phonologically plausible provided:
  - its markedness pattern does not have cardinality 3
  - its markedness pattern is not the singleton $\{\langle 0, 0\rangle\}$
- For each phonologically plausible markedness patterns and each sets of form (with some mild technical restrictions), construct the corresponding typology $\Longrightarrow$ a total of roughly 150 languages
- There are only 6 languages that are $\mathcal{F}$-relevant! here I list those that require F_2 above F_1 (the others are analogous by feature symmetry)

markedness pattern μ	target language L
$\mu = \{\langle 0, 0\rangle, \langle 1, 1\rangle\}$	$L = \left\{ \langle 0, 1\rangle \; \langle 1, 0\rangle \right\} \cup \left\{ \langle 0, \#\rangle \; \langle \#, 0\rangle \; \langle \#, 1\rangle \right\}$
$\mu = \{\langle 0, 1\rangle, \langle 1, 0\rangle\}$	$L = \left\{ \langle 0, 0\rangle \; \langle 1, 1\rangle \right\} \cup \left\{ \langle 0, \#\rangle \; \langle \#, 0\rangle \; \langle \#, 1\rangle \right\}$
$\mu = \{\langle 0, 1\rangle\}$	
$\mu = \{\langle 0, 1\rangle\}$	$L = \left\{ \langle 0, 0\rangle \; \langle 1, 1\rangle \right\}$



- This shows that $\mathcal{F}$-irrelevant languages are the majority, as expected; Step 1 of the analysis of correctness is thus an interesting result

EDRAs on $\mathcal{F}$-relevant languages for plausible constraints

- Let's look for instance at the first one of these languages:

	F_1	F_2	M_1	M_2	M
⟨#,1⟩, ⟨#,0⟩		W		L	
⟨1,0⟩, ⟨0,0⟩	W		L		W
⟨1,0⟩, ⟨0,1⟩	W	W	L	W	
⟨0,1⟩, ⟨0,0⟩		W		L	W
⟨0,1⟩, ⟨1,0⟩	W	W	W	L	

- Data that have a W corresponding to M will trigger at most one update and can thus be effectively discarded:

	F_1	F_2	M_1	M_2	M
⟨#,1⟩, ⟨#,0⟩		W		L	
⟨1,0⟩, ⟨0,1⟩	W	W	L	W	
⟨0,1⟩, ⟨1,0⟩	W	W	W	L	

- If the fist datum is fed at least once, the EDRA wil get right the desired ranking F_2 above F_1
- Analogous considerations hold for the other $\mathcal{F}$-relevant languages

Step 3 of correctness: toy version, and beyond

Theorem (Toy version of step 3 of the analysis of correctness)

Within the toy framework for OT segmental phonotactics considered, EDRAs are correct on $\mathcal{F}$-relevant languages provided (binary markedness) constraints are phonologically plausible

Now I need to scale up this little case study:

- assemble a very large constraint set from the OT literature:
  - lots of features
  - markedness constraints that target more than two features
  - lots of feature interaction (interaction graph)
- extract $\mathcal{F}$-relevant languages from the corresponding typology
  - by developing results on licit immediate rankings in minimal generating partial rankings
- study behavior of EDRAs on those languages
  - by developing proper tools for the analysis of EDRAs